Math Explorations

A Serious Series

I created this problem mucking about with fractions while teaching a class in rational numbers at the University at Buffalo. When I saw the pattern of the sums, it literally made me laugh with pleasure! The students were mostly math majors and really, really wanted an algebraic solution, which is possible (see the end of the problem unless you want to solve it on your own).

Mathematical Series - the sum of a list of numbers which are generated according to a pattern or rule.

When encouraging students to explore patterns in series, suggest they build the series slowly, first with one term, then two terms, then three terms, and so on. This particular pattern is delightful and should jump right out at them if they are orderly in their exploration

Consider $\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+\frac{1}{4x5}+\frac{1}{5x6}$

Express the sum as a fraction in lowest terms.

Possible questions to guide a discussion

- What do you see?
- Are there fractions that might fit to the right or the left of the series?
- How many fractions could be included if this series continued?
- What is the least number the answer might be? What is the greatest?
- What would the sum be if you only had two, three, or four fractions in the series?
- Did you see any patterns when you were adding the fractions?
- Did you think differently about common denominators when adding these fractions?
- What would be the sum of the first 10 fractions in this series?
- What would be the sum of the first 25 fractions in this series?
- What would be the sum of the first 100 fractions in this series?
- What number is this series trying to get close to?
- Will this series ever exceed 1 if it went forever?
- Do you think there will always be a pattern in math problems?
- What other math series does this make you curious about?
- How did you feel when you saw something new or interesting when working on this problem?

I much prefer the slow building up of the series by beginning with one term, two terms, three terms and so on. The pattern explodes in your face that way, but for those who are wed to algebra, read on.

Spoiler alert! The next page shows an algebraic assist for determining the solution.

Algebraic assist for finding the sum

$$\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+\frac{1}{4x5}+\frac{1}{5x6}$$

By decomposition of fractions

$$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

You can continue with the algebraic solution or combine it with numeric substitution.

Replace each fraction in the original series with the above equality

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$$

I will leave it for you to finish. It would be cruel to take all the fun away.