Math Explorations

Using a Skills Trajectory to Efficiently Target Misconceptions and Remedy Errors

E.T. Walker (2020)

Our students are required to process, categorize, and make sense of an enormous amount of information every day. Some of that knowledge provides a foundation for future learning. Chances are high that a student will be absent, distracted, or hear something the teacher says incorrectly multiple times during the day. The result is an incomplete or misconstrued knowledge stream that can come back to bite the student time and again.

In mathematics, finding those small misconceptions or missing pieces is difficult, especially in whole number, fraction, and decimal computation. Classroom assessments are not created to identify small errors and most teachers do not have the time to examine student work for repeated misconceptions. Errors are marked as simply incorrect. Students replicate the same mistakes and experience poor or failing grades. Fear and dislike of mathematics begins to build.

Skills trajectories offer an efficient and effective way to go back to the beginning of a skill, identify those points where learning was interrupted, and quickly provide targeted remediation. Here is how they work.

Think about learning to tie your shoes. Most likely you can tie your shoes without looking or thinking of the individual steps. This skill has not always been in your repertoire. You learned it painstakingly step by step. Someone broke those steps down for you and taught you each one. Go ahead, try to list each step for tying a shoe. This is the deconstruction of a final skill.

A Skills Trajectory completes the same process for successful completion of a math skill. Consider addition of whole numbers. Here is a possible Skills Trajectory for the end goal of multi-digit addition.

<u>Skills Trajectory</u>
<u>Multi-Digit Addition for Whole Numbers</u>

<u>Kindergarten begins here</u>

Assumptions

- ✓ Understanding of the concept of addition (putting together)
- ✓ Showing addition as combining groups using manipulatives or models

Level 1 - Counting on within 100. I say 14, you say the next number. I say 21 you say the next 2 (or 3) numbers both within tens and across tens.

Level 2 - Adding single digits within 10 both horizontally and vertically with or without manipulatives (fingers are fine)

1st grade begins here

Level 3 - Two-digit + one-digit, no regrouping

Level 4 – Two-digit + two-digit, no regrouping

Level 5 - Two-digit + one-digit, regrouping one's place

Level 6 – Two-digit + two-digit, regrouping one's place

2nd Grade begins here

Level 7 – Two-digit + two-digit, regrouping ten's place, only

Level 8 – Two-digit + two-digit, regrouping BOTH one's and ten's place

Level 9 – Three-digit + one-digit with and without regrouping

Level 10 – Three-digit + two-digit, with and without regrouping one place value

Level 11 – Three-digit + three-digit, no regrouping

Level 12 – Three-digit + three-digit, regrouping one's or ten's place

3rd Grade begins here

Level 13 – Three-digit + three-digit, regrouping two place values

Level 14 – Three- or four-digit + three-digit, regrouping into the thousand's place

This trajectory or path has 14 levels. It is easy to create an assessment with items matching each level to see if there is an obvious break in knowledge or a stumbling block to successfully adding. If a whole class or group is given this diagnostic assessment, students can be divided into study groups for addressing misconceptions.

Here is a sample assessment for the first six levels. If all students begin with Item 1, errors will be evident even if they fall before a student's current grade level. Using a tracker, student work for each item is recorded as either completely correct using a code of (1) or incorrect in any way using a code of (0).

Teachers can sort assessments by errors to create a toolbox of error healing strategies when working with students.

Pre and Post Assessment for Adding Multi-Digit Numbers

Name ____ Date _____________________________

1. Count forward: 45 ,____,____, ____, 49	2. Add: 4 + 3 = ______	3. Add: 1 2 + 3
4. Add: 1 4 + 2 1	5. Add: 2 7 + 5	6. Add: 4 6 + 2 7
Additional items would follow for the remaining levels of the trajectory.		

Experience with this Skills Trajectory showed that 5th graders, who struggled with adding decimals, understood the concept of place value alignment but where missing skills related to regrouping which could be traced back to 2nd grade! How painful math must have been for these students during the ensuing years of study. Remediating these errors allowed students to be successful when adding decimals.

Once a break in the trajectory has been identified, practice sheets of each individual step can be used as teaching tools. These practice sheets become an ideal place for students to engage in math talks, comparing and contrasting answers and techniques for computing. Teachers are free to instruct in the manner that works best for their students using the tools that they deem most helpful for cognitive understanding and skill development.

Students and teachers can track progress, which the environment of online games has made highly desirable as students "level up".

Finally, students retake the initial item assessment to make sure learning has been sticky. Any remaining errors can be revisited. The time saved in diagnosing errors and the transition to math-able students is a win-win for all parties!